Investment timing, collateral, financing constraints

Takashi Shibata
Tokyo Metropolitan U.

joint work with Michi Nishihara at Osaka U.

March 23, 2017
LabEx ReFi

Previous papers about financing constraints

Standard model: no financing constraints

1. McDonald and Siegel (1986 QJE): all-equity financing
2. Sundaresan and Wang (2007 AER): debt-equity financing

Equity financing constraints model:

3. Boyle and Guthrie (2003 JF), Nishihara and Shibata (2013, JBF):
    - Investment thresholds have a V-shaped curve with frictions.

Debt financing constraints model:

4. Shibata and Nishihara (2012 JBF; 2015 EJOR; 2015 JBF):
    - Investment thresholds have a U-shaped curve with frictions.
    - Upper limits depend on *investment quantity-based constraints*.
5. This presentation:
    - Upper limits depend on *collateral-based constraints*.

We examine how *collateral-based debt financing constraints* influence the interaction between investment and financing decisions.

Outline

1. Model setup and value functions
2. *Exit (default and shutdown)* strategies
3. *Entry (investment and financing)* strategies
   - (P1) Unlevered (all-equity financed) firm:
     McDonald and Siegel (1986, QJE)
   - (P2) Unconstrained levered (debt-equity financed) firm:
     Sundaresan and Wang (2007, AER)
   - (P3) Constrained levered (debt-equity financed) firm:
     our model
4. Model implication
5. Concluding remarks

Model setup

- A firm possesses an investment opportunity
- $qX(t)$: cash flow after investment
  - $q \geq 0$: quantity,
  - $X(t)$: price

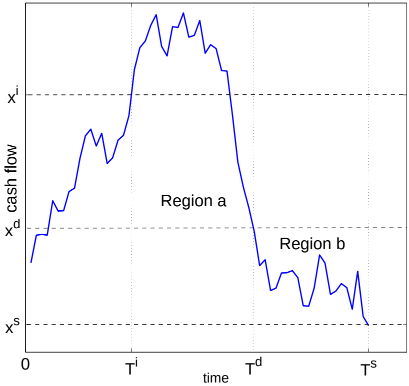
$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}z(t),\ X(0) = x > 0, \tag{1}$$

  where $\mu \in (0, r)$, $\sigma > 0$, $(z(t))_{t\geq 0}$: standard Brownian motion, and $r > 0$: risk-neutral discount factor.
- $I(q) > 0$: one-time fixed investment cost where

$$I(0) > 0, \quad I'(q) > 0, \quad I''(q) > 0.$$

- $kI(q) > 0$: residual value at liquidation ($k \in [0, 1]$), i.e., collateral
  - $k = 0$: completely *irreversible*
  - $k \in (0, 1)$: *partially reversible*
  - $k = 1$: completely *reversible*
- Debt-equity financing
  - A firm issues a *perpetual debt* (for simplicity).

Debt-equity financing and collaterals



- x^{i}:
  investment threshold
- x^{d}:
  default (bankruptcy) threshold
- x^{s}:
  shutdown (liquidation) threshold

- $T^{\mathrm{i}} := \inf\{u \geq 0 \mid X(u) \geq x^{\mathrm{i}}\}$: *investment time*
- $T^{\mathrm{d}} := \inf\{u \geq T^{\mathrm{i}} \mid X(u) \leq x^{\mathrm{d}}\}$: *default (bankruptcy) time*
- $T^{\mathrm{s}} := \inf\{u \geq T^{\mathrm{d}} \mid X(u) \leq x^{\mathrm{s}}\}$: *shutdown (liquidation) time*

Equity value under all-equity financing (indexed by "0")

$$E_0^{\mathrm{a}}(X(t), q_0) \tag{2}$$
$$= \sup_{T_0^{\mathrm{s}}} \mathbb{E}^{X(t)} \left[\int_t^{T_0^{\mathrm{s}}} \mathrm{e}^{-r(u-t)} \underbrace{(1-\tau) q_0 X(u)}_{\text{cash flow after tax}} \mathrm{d}u + \mathrm{e}^{-r(T_0^{\mathrm{s}}-t)} \underbrace{(1-\alpha) k I(q_0)}_{\text{residual value}} \right],$$

for any $t > T_0^{\mathrm{i}}$ (i.e., after investment), where

- superscript "a" represents the value *after investment*
- subscript "0" represents the *unlevered (all-equity financed)* firm.
- $\tau > 0$: tax rate
- $k \in [0, 1]$: proportion of reversible investment
- $\alpha k I(q_0)$: bankruptcy cost where $\alpha \in (0, 1)$

Equity value after investment under all-equity financing:

$$E_0^{\mathrm{a}}(X(t), q_0) = vq_0X(t) + \Big(\underbrace{(1-\alpha)kI(q_0)}_{\text{residual value}} - vq_0 x_0^{\mathrm{s}}(q_0)\Big)\left(\frac{X(t)}{x_0^{\mathrm{s}}(q_0)}\right)^{\gamma}, \quad (3)$$

where

$$v := \frac{1-\tau}{r-\mu} > 0, \quad \gamma := \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\Big(\frac{\mu}{\sigma^2} - \frac{1}{2}\Big)^2 + \frac{2r}{\sigma^2}} < 0.$$

Optimal shutdown threshold (maximizing (3) with x_0^{s} gives):

$$x_0^{\mathrm{s}}(q_0) = \lambda(1-\alpha)k\frac{I(q_0)}{q_0} > 0, \quad (4)$$

where $\lambda := \gamma/((\gamma-1)v) > 0$.

Value functions under debt-equity financing

Debt is risky if

$$\underbrace{\frac{c}{r}}_{\text{principal of debt}} > \underbrace{(1-\alpha)kI(q),}_{\text{residual value at liquidation}} \tag{5}$$

for fixed c and q.

Risky debt and riskless debt:

Debt is risky if $k \in [0, k_2(c, q))$ for a fixed c and q, where

$$k_2(c, q) := \min\left\{1,\ \inf\left\{k \geq 0 \;\middle|\; k \geq \frac{c}{r(1-\alpha)I(q)}\right\}\right\}, \tag{6}$$

for a fixed c and q.
Debt is riskless otherwise ($k \in [k_2(c, q), 1)$).

Equity value after investment: $E^{\mathrm{a}}(X(t), c, q)$

$$E^{\mathrm{a}}(X(t), c, q) = \sup_{T^{\mathrm{d}}} \mathbb{E}^{X(t)} \left[\int_t^{T^{\mathrm{d}}} \mathrm{e}^{-r(u-t)} \underbrace{(1-\tau)(qX(u)-c)}_{\text{cash flow after tax}} \mathrm{d}u \right], \qquad (7)$$

for any $t > T^{\mathrm{i}}$ (i.e., after investment), where

- $k \in [0, k_2(c, q))$, i.e., debt is risky
- superscript "a" represents the value *after investment*

Equity value after investment for a levered firm:

$$E^{\mathrm{a}}(X(t),c,q) = vqX(t) - \frac{1-\tau}{r}c - \left(vqx^{\mathrm{d}}(c,q) - \frac{1-\tau}{r}c\right)\left(\frac{X(t)}{x^{\mathrm{d}}(c,q)}\right)^{\gamma}, \quad (8)$$

Optimal default (operating concern bankruptcy) thresholds:

$$x^{\mathrm{d}}(c,q) = \lambda\frac{1-\tau}{r}\frac{c}{q} \geq 0, \quad (9)$$

- $x^{\mathrm{d}}(c,q)$ is linear function of c/q

Debt value after investment: $D^{\mathrm{a}}(X(t), c, q)$

$$D^{\mathrm{a}}(X(t), c, q) \tag{10}$$
$$= \mathbb{E}^{X(t)}\left[\mathrm{e}^{rt}\left(\int_t^{T^{\mathrm{d}}(c,q)} \mathrm{e}^{-ru} c \mathrm{d}u + \mathrm{e}^{-rT^{\mathrm{d}}(c,q)}(1-\alpha)\underbrace{E^{\mathrm{b}}(x^{\mathrm{d}}(c,q),q)}_{\text{equity value after bankruptcy}}\right)\right],$$

for any $t > T^{\mathrm{i}}$, where

$$E^{\mathrm{b}}(x^{\mathrm{d}}(c,q),q) \tag{11}$$
$$= \sup_{T^{\mathrm{s}}(q)} \mathbb{E}^{x^{\mathrm{d}}(c,q)}\left[\mathrm{e}^{rT^{\mathrm{d}}(c,q)}\left(\int_{T^{\mathrm{d}}(c,q)}^{T^{\mathrm{s}}(q)} \mathrm{e}^{-ru}(1-\tau)qX(u)\mathrm{d}u + \mathrm{e}^{-rT^{\mathrm{s}}(q)} kI(q)\right)\right],$$

- superscript “b” represents the value function *after bankruptcy*.

Debt value after investment:

$$D^{\mathrm{a}}(X(t),c,q)=\frac{c}{r}-\left(\frac{c}{r}-(1-\alpha)\underbrace{E^{\mathrm{b}}(x^{\mathrm{d}}(c),q)}_{\text{equity value after bankruptcy}}\right)\left(\frac{X(t)}{x^{\mathrm{d}}(c,q)}\right)^{\gamma},\quad (12)$$

where

$$E^{\mathrm{b}}(x^{\mathrm{d}}(c),q)=\begin{cases} vx_1^{\mathrm{d}}(c)+\left(\underbrace{kI(q)}_{\text{residual value}}-vqx^{\mathrm{s}}(q)\right)\left(\frac{x^{\mathrm{d}}(c,q)}{x^{\mathrm{s}}(q)}\right)^{\gamma}, & k\in[0,k_1(c,q)), \\ kI(q), & k\in[k_1(c,q),k_2(c,q)], \end{cases}\quad (13)$$

where $x^{\mathrm{s}}(q)=\lambda kI(q)/q$ and

$$k_1(c,q):=\min\left\{1,\inf\left\{k\geq 0\ \middle|\ k\geq(1-\tau)\frac{c}{r}\right\}\right\}.\quad (14)$$

- Note that $x^{\mathrm{s}}(q)\geq x_0^{\mathrm{s}}(q)$

Summary of exit strategies

Proposition 1:

For a fixed $c \geq 0$ and $q \geq 0$, we obtain

	$k \in [0, k_1)$	$k \in [k_1, k_2]$	$k \in (k_2, 1]$
	risky debt		riskless debt
Default	equity holders $x^{\mathrm{d}}(c, q)$	equity holders $x^{\mathrm{d}}(c, q)$	–
Shutdown	debt holders $x^{\mathrm{s}}(q)$		
Exit strategies	sequential	simultaneous	

(P1) Unlevered (all-equity financed) firm

(P1):

$$E_0^*(x) = \left(\max_{x_0^{\mathrm{i}}, q_0} \; H_0(x_0^{\mathrm{i}}, q_0) \right) x^{\beta} \tag{15}$$

where

$$H_0(x_0^{\mathrm{i}}, q_0) := {x_0^{\mathrm{i}}}^{-\beta} \left\{ E_0^{\mathrm{a}}(x_0^{\mathrm{i}}, q) - I(q) \right\}. \tag{16}$$

- $\beta := 1/2 - \mu/\sigma^2 + \sqrt{(\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2} > 1$.
- superscript "$*$" indicates the *nonconstrained* optimum.
- $(x_0^{\mathrm{i}*}, q_0^*) = \mathrm{argmax}_{x_0^{\mathrm{i}}, q_0} \; H_0(x_0^{\mathrm{i}}, q_0)$.

(P2) Unconstrained levered (debt-equity financed) firm

(P2):

$$E^{*}(x) = \left(\max_{x^{\mathrm{i}}, c, q} H(x^{\mathrm{i}}, c, q) \right) x^{\beta}, \tag{17}$$

where

$$H(x^{\mathrm{i}}, c, q) := x^{\mathrm{i}^{-\beta}} \left\{ E^{\mathrm{a}}(x^{\mathrm{i}}, c, q) + D^{\mathrm{a}}(x^{\mathrm{i}}, c, q) - I(q) \right\}. \tag{18}$$

- superscript "$*$" indicates the *nonconstrained* optimum.
- $(x^{\mathrm{i}*}, c^{*}, q^{*}) = \mathrm{argmax}_{x^{\mathrm{i}}, c, q} H(x^{\mathrm{i}}, c, q)$.

(P3) Constrained levered (debt-equity financed) firm

(P3):

$$E^{**}(x) = \left(\max_{x^{\mathrm{i}}, c, q} H(x^{\mathrm{i}}, c, q) \right) x^{\beta}, \quad (19)$$

$$\text{subject to } D^{\mathrm{a}}(x^{\mathrm{i}}, c, q) \leq \phi \underbrace{kI(q)}_{\text{residual value}} . \quad (20)$$

- superscript "$**$" indicates the *constrained* optimum.
- $k \in [0, 1]$: partial reversibility parameter
- $\phi \in [0, 1]$: financial friction parameter
- $kI(q)$: residual value (i.e., collateral)
- $(x^{\mathrm{i}**}, c^{**}, q^{**})$
  $= \mathrm{argmax}_{x^{\mathrm{i}}, c, q} H(x^{\mathrm{i}}, c, q)$ subject to $D^{\mathrm{a}}(x^{\mathrm{i}}, c, q) \leq \phi kI(q)$.

Our intuitive conjecture

Proposition 2 (Unconstrained problem "$*$"):

We have $x^{i*} \leq x_0^{i*}$ because of "leverage effect."

- This is the same as in SW (2007, AER).

Intuitive conjecture (Constrained problem "$**$"):

$$x^{i*} \leq x^{i**} \leq x_0^{i*} \quad \text{for a fixed } \phi \text{ and } k. \tag{21}$$

- "$x^{i*} \leq x^{i**}$:" *financing constraints delay the investment.*
- "$x^{i**} \leq x_0^{i*}$:" *debt financing accelerates the investment, even under constraints.*
- These inequalities of (21) are *not always* correct...

Numerical examples

Cost function:

$$I(q) = F + q^2, \text{ where } F > 0.$$

Parameters:

$$r = 9\%, \sigma = 10\%, \mu = 0\%, F = 5, \tau = 15\%, \alpha = 40\%, \phi = 1, x = 0.2,$$

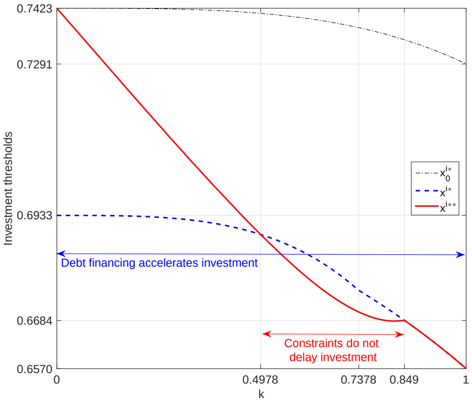
and

$$k \in [0, 1] \quad \text{for } \phi = 1$$
$$\phi \in [0, 1] \quad \text{for } k = 1$$

Then we have

$$k\phi \leq 1.$$

Entry (investment) strategies with k



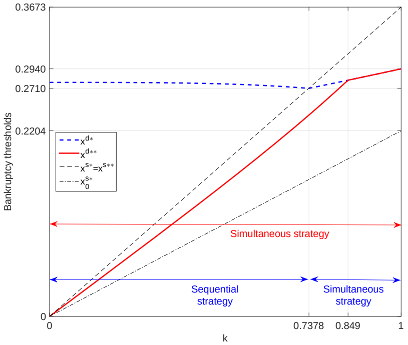
Observation 1

$x^{i*} \leq x^{i**} \leq x_0^{i*}$
is *not always* correct.

Economic implication of Observation 1

Financing constraints accelerate the investment for $k \in (0.67, 0.94)$.

Exit (bankruptcy) strategies with k



Observation 2

Unconstrained firm:

$x^{d*} \geq x^{s*}$ for $k \leq 0.73$.

$x^{d*} < x^{s*}$ for $k \in (0.73, 1]$.

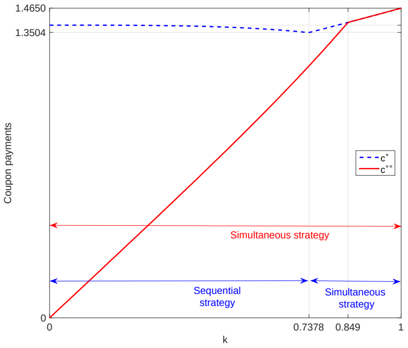
Constrained firm:

$x^{d**} < x^{s**}$ for any k.

Economic implication of Observation 2

- *Unconstrained firm exercises* default and shutdown sequentially for $k \leq 0.73$, while it does them simultaneously for $k \in (0.73, 1]$.
- *Constrained firm exercises* default and shutdown simultaneously for any $k \in [0, 1]$.

Coupon payments with k



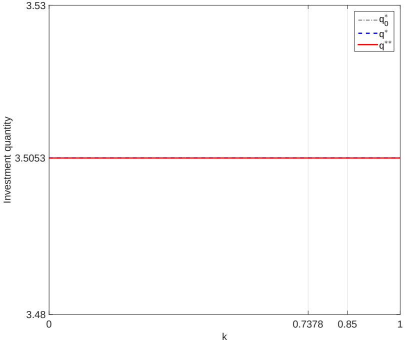
Observation 3

$c^{**} \in [0, c^*]$ for any $k \in [0, 1]$.

Economic implication of Observation 3

- *Unconstrained* coupon payment is *decreasing* with $k \leq 0.73$, while it is *increasing* with $k \in (0.73, 1]$.
- *Constrained* coupon payment is *increasing* with $k \in [0, 1]$.

Investment quantity with k



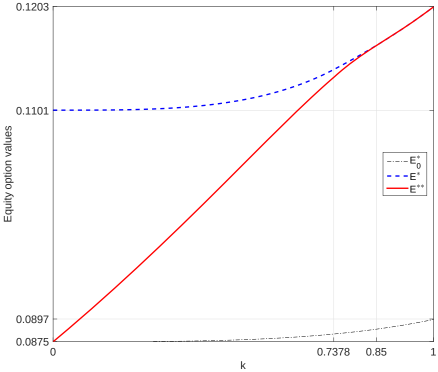
Observation 4

$q^{**} = q^{*} = q_0^{*}$.

Economic implications of Observation 4

Constrained investment quantity is *constant* with k.

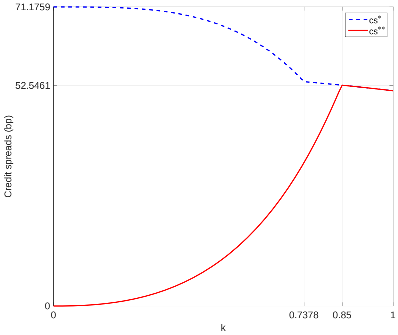
Option values with k



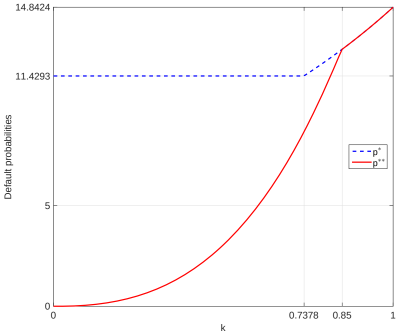
Observation 5

Constrained option value is *monotonically increasing* with k.

Credit spreads and default probabilities with k



Credit spreads



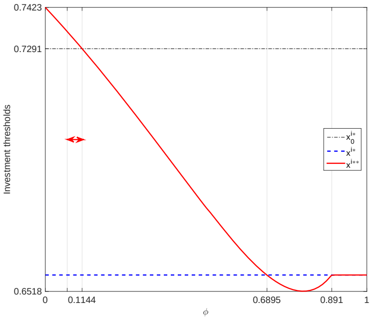
Default probabilities

Observation 6

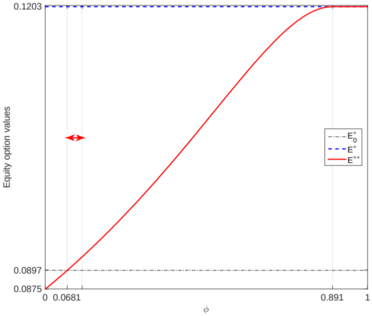
Constrained spreads and probs. are smaller than *nonconstrained ones.*

Financing constraints decrease credit spreads and default probs.

Effects of friction ϕ



Investment thresholds



Equity option values

Observation 7

For $\phi \in (0.0681, 0.1144)$, $x^{i**} > x_0^{i*}$ and $E^{**}(x) > E_0^*(x)$.

We always have $x^{i*} \le x_0^{i*}$ and $E^*(x) \ge E_0^*(x)$ for any ϕ.

Implication of collateral model

No constraint model:

$$x^{i*} \leq x_0^{i*} \text{ and } E^*(x) \geq E_0^*(x), \quad \phi \in [0,1]$$

Unconstrained debt financed firm always *accelerates* the investment, compared with *all-equity financed firm*.

Constraint model:

$$x^{i**} > x_0^{i*} \text{ and } E^{**}(x) > E_0^*(x), \quad \phi \in (0.0681, 0.1144)$$

Constrained debt financed firm may *delay* the investment, compared with *all-equity financed firm*.

Concluding remarks

Main results:

- $x^{i*} \leq x^{i**} \leq x_0^{i*}$ is *not always* correct.
  (although $0 \leq c^{**} \leq c^*$ and $q_0^* = q^{**} = q^*$).
- $E^{**}(x) > E_0^*(x)$ and $x^{i**} > x_0^{i*}$ may be obtained.
  (although $E^*(x) \geq E^*(x)$ and $x^{i*} \leq x_0^{i*}$).
- $cs^{**} \leq cs^*$ and $p^{**} \leq p^*$ are always obtained.

Economic implications of main results:

- *Constrained levered* firm may *accelerate* the investment, compared with *unconstrained levered* firm.
- *Constrained levered* firm may *delay* the investment, compared with unlevered firm.
- *Debt financing constraints decrease credit spreads* and *default probabilities* (irregardless of they may accelerate the investment).