Optimal financing of highly innovative projects under double moral hazard

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Abstract

We model the investor-entrepreneur relationship involved in the financing of highly innovative projects under a double moral hazard setup. We show that a wide family of financing contracts are optimal, which fits well with mixed security structures observed in real-world such as convertible preferred equity, warrants and call options. Schemes that reward risk (and failure) are also desirable, as extreme returns can be good indicators of highly innovative investment projects. Numerical simulations show conditions for credit rationing to emerge when straight debt is used, stressing the welfare-improving role played by hybrid securities. As our results replicate several stylized facts of innovative firms, we argue that the proposed setup is a suitable starting point to model venture capital financing.

Keywords: reward for failure; optimal financial contract; entrepreneurship financing; convertible assets; non-monotone likelihood ratio property

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1 Introduction

High-tech innovative industries exhibit distinguishing features concerning their investment projects, as well as the schemes adopted to finance their ventures. First, investment projects of firms operating in this class of industries typically show a return distribution with: (i) abnormally high, but unlikely, success returns, and (ii) a high probability of failure.[1] Second, leverage in small and high growth technology-driven industries is lower than that of large and less innovative industries,[2] being frequent the use of hybrid financing structures that combine debt and equity, together with preferential seniority and conversion options, especially in the US.[3] Third, investors financing this type of business frequently provide the entrepreneur not only with funds, but also with effort by means of either advice, management, networks or monitoring.[4] Fourth, high-growth and more innovative firms seem to face greater barriers to external finance, which suggests that credit rationing may be a phenomenon relatively more prevalent in this class of firms.[5]

As an attempt to account for these features and stylized facts, this paper proposes a financial contracting model that formalizes the relationship between an investor and an entrepreneur when investment projects with the above described characteristics are involved. To that end, our setting departs jointly from two traditional assumptions in contract theory. First, in order to model highly innovative projects, we assume a nonmonotone statistical relationship between (verifiable) returns and (unverifiable) innovation, which technically implies departing from the conventional monotone likelihood ratio property (MLRP). Second, in order to allow the investor to also influence innovation decisions, we assume a two-side moral hazard setup so that both the principal and the agent must be incentivized to attain an optimal joint innovation level. We then fully characterize the optimal financing schemes of highly innovative investment projects under a double moral hazard setup.

Our results point out that the two-side moral hazard nature of our framework implies that the first-best and second-best contracts involve a balanced solution, so that a positive innovation level of *both* the investor and the entrepreneur is needed. Under the symmetric information environment, the efficient innovation level can be implemented

[1] Empirical evidence suggests that the fraction of successful innovative investment projects is typically not superior to 20% (Bergemann and Hege, 1998; Sahlman, 1990; Gorman and Sahlman, 1989).

[2] See Chang and Song (2014), Frank and Goyal (2003), Barclay et al. (1995), and Long and Mantz (1985).

[3] See Kaplan and Stromberg (2003), Kaplan et al. (2012), Bengtsson and Sensoy (2011), Sahlman (1990), and Trester (1998).

[4] See Kaplan and Stromberg (2004), Cumming and Johan (2007), Sapienza et al. (1996), Gorman and Sahlman (1989), and Bengtsson and Sensoy (2011).

[5] See Chang and Song (2014), and Brown (1997).

with either a full-insurance or a full-franchise contract. Interestingly, however, both types of contracts have to incorporate a penalization: against the entrepreneur in the former scheme and against the investor in the latter one.

Under the asymmetric information environment, we show that a wide family of financing rules can achieve the optimal (second-best) innovation level, including strictly increasing, strictly decreasing, and a wide variety of other nonmonotone schemes. As a consequence of the double moral hazard problem (reminiscent of team moral hazard), though, there is no financing rule under asymmetric information that be able to implement first-best innovation levels.

The wide variety of nonmonotone optimal contracts under asymmetric information is the consequence of highly innovative entrepreneurial business, as with this class of projects it is not the shape (average slope) of the contract which matters, but the relationship between the rewards of extreme returns. Technically, this phenomenon occurs because when the MLPR assumption regarding observed returns and unobserved innovation decisions is not satisfied, not only success but also failure can be a good signal of truly innovative projects. Consequently, we show that a salient class of optimal nonmonotone financing schemes are those that reward extreme returns and punish, in relative terms, moderate returns, such as the U-shaped and J-shaped contracts. These risk-reward schemes also involve a reward for failure property, which although counter-intuitive, can be associated to hybrid contracts used by entrepreneurial business to finance their highly innovative projects in practice.

In the particular case of the strictly decreasing financing rule, its optimally is a consequence of both the no MLRP assumption and the double moral hazard setup. Under this setting, a scheme may reward extreme results by optimally combining the incentives provided to both the investor and the entrepreneur. A decreasing scheme can then work if the relative reward given to the investor (entrepreneur) exceeds the relative penalization imposed to the entrepreneur (investor) when highest (lowest) returns are observed. That is, since the ultimate goal of the financial rule is the joint innovation, a decreasing scheme can be optimal if it properly counterbalances the opposite incentives generated over the two parties of the relationship.

We demonstrate that the features of various of these theoretical optimal contracts are consistent with nonlinear and hybrid financing schemes —which combine equity and other securities— used in practice to fund investment projects with high-return and high-risk. Specifically, we explore implementation issues related to two optimal financing schemes. First, we study the successful implementation of a strictly increasing contract through convertible preferred equity and a sequence of warrants. Second, we examine the implementation of a risk-reward contract (the U-shaped scheme) via equity plus either call options or a dilution process in favor of the investor. In the latter case, we analyze the possibility that the investor can infuse new capital when returns are sufficiently low, which allows her to cross-subsidize incomes stemming from success and failure scenarios. All these optimal hybrid structures highlight the importance of combining both inside and outside equity, property that ensures, under a double moral hazard setting, the provision of high-powered incentives to undertake innovation from both the investor and the entrepreneur.

In addition, we study two classes of suboptimal contracts which are pertinent in the context of the model: the straight debt and the full-franchise contract. In the first case, our analysis suggests that the debt scheme induces a corner solution so that only the entrepreneur undertakes innovation. In the context of a double moral hazard environment, this finally leads to a joint innovation and an expected surplus lower than those achieved with the optimal contract. This result follows from the inability of the pure debt scheme to replicate a crucial property of our optimal hybrid security, namely, giving equity to the investor and the entrepreneur, and thus, providing both parties with high-powered incentives to undertake innovation. Interestingly, this theoretical result is consistent with the evidence above cited concerning the fact that leverage is less frequent in high-tech innovative industries. Further, comparative statics exercises reveal that the debt contract admits the possibility of *credit rationing* over profitable investment projects with either (i) a mass of probability sufficiently concentrated on moderate returns, or (ii) a probability of failure too large. This implies that our optimal mixed securities are particularly welfare enhancing when financing investment projects with any of these two characteristics. This is because these projects, despite to be profitable, may not be funded under conventional debt schemes such as bank loans or bond issues. This implication of the model is also supported by the evidence mentioned at the beginning of the paper, as more innovative firms seem to be particularly financially constrained.

In the case of the full-franchise contract, despite extant literature has shown that this class of scheme attains the first-best effort level under risk-neutrality and single moral hazard, we prove that it not only no longer holds under double moral hazard, but also that this scheme is neither a second-best contract. This occurs because the franchise scheme performs similarly to the debt contract in terms of innovation (a corner solution), as it is neither able to highly motivate to both partners of the agency relationship.

An important contribution of our work is to characterize innovation as an endogenous variable which depends on primitive characteristics of the investment projects, such as the parameters concerning the space and the probability distribution of their returns. To illustrate this property, we carry out different static comparative exercises over these parameters and the effects of their changes on the optimal contracts. Three classes of results emerge from these numerical simulations. First, these simulations show that optimal (first and second-best) innovation levels and expected surplus will increase if investment projects have either (i) a probability mass less concentrated on moderate returns —a higher level of probability extremism— or (ii) a larger level of abnormally good returns —a higher return skew towards success—. In contrast, optimal innovation and surplus will decrease if projects have a probability mass more concentrated on bad returns —a higher probability skew towards failure—. Second, our simulations also suggest which type of projects suffer larger costs associated to asymmetric information, by means of the computing of innovation and surplus gaps between the first and second-best solutions. Finally, the numerical exercises regarding parameters of the return distribution allow us to perform two additional analysis: (i) to identify a priori a project in accordance with a three-class typology established by the industry devoted to fund innovative ventures and predict its performance; and (ii) to examine the impact of technological shocks on the innovation levels and the social value created by highly innovative investment projects.

The rest of this paper is organized as follows. Section 2 reviews the closer previous research, identifying the contributions of our paper by means of a comparison of it with such literature. Section 3 proposes a model of financing of highly innovative investment projects under double moral hazard. Section 4 characterizes fully the family of optimal financial contracts (first and second-best), with special emphasis on those schemes that involve a reward for risk-taking feature, and thereby, a reward for failure property. Section 5 performs various comparative static analyses on optimal innovation and surplus, and their relationship with different parameters of the return distribution. A similar analysis is also applied to a specific case of the optimal risk-reward schemes, the U-shaped contract. Section 6 explores and compares different alternatives to implement the optimal financing scheme via straight debt, franchise contracts, and hybrid securities structures such as convertible preferred equity, warrants, and call options. Finally, Section 7 discusses the main conclusions. Most of the proofs are contained in the Appendix.

2 Related literature

This paper is related to previous literature on optimal financial arrangements under a double-sided moral hazard environment, especially that devoted to venture capital financing. In general, one of the main goals of this literature is to show and justify the optimality of mixed securities when financing this class of entrepreneurial business. In this vein, Casamatta (2003) adopts a single stage financing in which effort and advice are substitutes and outside financial investment is endogenous, showing that it is optimal to give preferred stocks to the venture capitalist (entrepreneur) when the level of outside financing is sufficiently high (low). Repullo and Suarez (2004) assumes a two-stage financing setting (start-up and expansion stages) in which effort and advice are complements, concluding that a set of standard non-linear claims (warrants) are optimal when interim profitability of the project is not verifiable. Schmidt (2003) adopts an incomplete contract approach where entrepreneur and investor's efforts are sequential decisions and can be either substitutes or complements, showing that convertible debt outperforms any standard debt-equity contract since it induces first-best decisions in each state of the world. Wang and Zhou (2004) proposes a two-period setup in which entrepreneur effort and outside investment resemble a double moral hazard environment, demonstrating that equity sharing and staged financing (instead of upfront financing) work complementarily to achieve approximately the first best for high-potential ventures. Inderst and Müller (2004) propose an equilibrium model in which investor and entrepreneur efforts can be substitute or complementary, showing that a combination of debt, inside and outside equity optimally balances the incentives of both agents when their bargaining powers depend ultimately on primitive market characteristics.

Unlike our work, none of these previous papers, however, model what really means for a firm to be highly innovative regarding the return distribution of its projects. In particular, all these works do not depart from the classic MLRP, and thus they assume that the entrepreneur and investor actions influence monotonically and positively the return distribution, without linking innovative efforts to either more riskiness, heavier tails or other statistic moments different from the expected return.

This paper also has connections with theoretical works, which although adopt either single moral hazard or no moral hazard at all, also provide explanations for the use of hybrid securities to financing innovative firms. This literature argues that non-standard claims are optimal because they (i) balance properly the venture capitalist's intervention incentives and the entrepreneur's control desire coming from debt and equity (Marx, 1998), (ii) reduce the entrepreneur incentives to focus on short-term success and window dressing activities (Cornelli and Yosha, 1997), (iii) mitigate distributional-control-manager replacement conflicts associated with a future sale of the firm (Berglöf, 1994), and (iv) allow the entrepreneur to solve a tension between two dimensions of moral hazard, namely, effort and excessive risk-taking (Biais and Casamatta (1999).

Finally, our paper is related to works showing theoretically conditions for credit rationing of start up and high-innovative firms (firms with profitable projects are not funded): In general this literature focus on the ole played by insufficient internal cash flow, and its relationship with: Biais and Casamatta (1999) (when tension between effort and risk taking entrepreneur's incentives is too strong but conditional on); Cornelli and Yosha (1997) (because staged financing and window dressing-short term bias). In contrast, we highlight the role played by the characteristics of the return's projects distribution of highly innovative firms, and in particular, its degree of skewness towards bad results (i.e. projects with abnormally high, but unlikely, returns and also high failure rates).

3 The Model

Consider the agency relationship between an investor (the principal, *she*) and the entrepreneur (the agent, *he*), regarding a project with an initial investment I normalized to zero. Since the entrepreneur has no initial wealth, all this investment is financed by the investor. Each the entrepreneur and the investor must choose an own innovation level through a simultaneous move game. The entrepreneur's innovation level is denoted by $a \in [0, 1]$ and the investor's one is represented by $p \in [0, 1]$.

Whereas a can be interpreted as operational or business innovation such as efforts expended on the production process, p can be though of as advise or managerial innovation such as marketing efforts. We then define $e = a + p$, the *joint* innovation level undertaken in this agency relationship, such that $e \in [0, 1]$.[6]

3.1 Innovation and return

We assume that although the entrepreneur's and the investor's innovation are private information, and thus, they are not verifiable by their counterpart, the return of the project is verifiable by both parties. Then, let $x_i^{(a,p)}$ be the return yield by a project with an innovation pair (a, p) when state of nature i occurs. Similarly, let $\pi_i^{(a,p)}$ be the conditional probability of observing return $x_i^{(a,p)}$, such that it is verified that $\pi_i^{(a,p)} > 0$ for all $i = 1, .., n$ and for all $(a, p) \neq (0, 0)$. Also, conditional probabilities are such that $\sum_{i=1}^{n} \pi_i^{(a,p)} = 1$ for all (a, p). These assumptions imply that, from observing a given x_i, it cannot be rule out a priori any innovation pair (a, p) with the exception of $(0, 0)$.

For simplicity, we assume that only three states of nature are possible $(i = 1, 2, 3)$, such that $x_i^{(a,p)} \in X = \{x_1, x_2, x_3\}$ for all (a, p), and $x_1 < x_2 < x_3$. Specifically, we

[6] We thus assume that a and p are substitutes, like Casamatta (2003). Other functional forms for joint innovation are also possible. In general terms, we can define $e = f(a, p)$, such that both parties' innovation levels can be productive and complementary with each other (see Kim and Wang, 1998; Repullo and Suarez, 2004).

suppose the following formulation for returns and their conditional distribution.

Assumption 1 (A1). The space of returns $x_i^{(a,p)}$ is

$$X = \{1 - \sigma, 1, k(1 + \sigma)\},$$

with $\sigma > 0$ and $k > 1$, and their probability distribution is described by[7]

$$\pi_i^{(a,p)} = \begin{cases} m\gamma(a+p) & \text{if } x_i = 1-\sigma \quad (i=1) \\ -\gamma(1+m)(a+p)+1 & \text{if } x_i = 1 \quad (i=2) \\ \gamma(a+p) & \text{if } x_i = k(1+\sigma) \quad (i=3) \end{cases},$$

with $m > 1$ and $\gamma \in \left(0, \frac{1}{1+m}\right)$.

This assumption deserves several comments.

Positive skewness. This formulation captures the idea that the entrepreneur faces an investment project whose return distribution exhibits a positive skewness, which is consistent with the empirical evidence on highly innovative ventures. This skewness is the consequence of assumptions on parameters k and m. Regarding the first parameter, notice that by assuming that $k > 1$ allows us to formalize the idea that the success return $x_3 = k(1 + \sigma)$ is potentially abnormally good, as it deviates more from the intermediate return $x_2 = 1$ than the failure return $x_1 = 1 - \sigma$. In statistical terms, this assumption ensures that the right tail of the return distribution is longer than its left tail. In turn, regarding the parameter m, notice that by assuming $m > 1$ allows us to formalize the idea that failure is much more likely that an outstanding, but unlikely, success. In statistical terms, this assumption ensures that the mass of the distribution is more concentrated on the left than on the right side of the distribution.

To illustrate how these two parameters characterize the asymmetry of the return distribution, let us take the intermediate return $x_2 = 1$ as a reference point. Then, if k and m were 1, the right and left sides of the distribution would be equally long and fat, and thus, the distribution would be symmetric. Accordingly, we will say that while the parameter k measures the project's *return skew towards success*, the parameter m measures the project's *probability skew towards failure*.

Type of projects. The formulation described by assumption (A1) is also consistent with empirical evidence which in general classifies innovative projects into three different profiles: (i) poor projects, (ii) living dead", and (iii) high flyers" (Sahlman, 1990;

[7]Biais and Casamatta (1999) propose a distribution function close to that analyzed here, but with binary effort and risk choices. This model doesn't study, however, investments projects with high-returns and high-risk. Loyola and Portilla (2014) also adopt a similar distribution function, but in the context of executive compensation. Nevertheless, neither of these two frameworks consider a double moral hazard setup.

Gorman and Sahlman, 1989). For instance, according to the sample studied by Salhman (1990), poor projects (35% of the sample) suffered a total loss or could not repay the initial investment. Living dead (50% of the sample) showed a moderate profitability, but venture capitalist in general did not invest additional resources or effort in this class of projects. Lastly, high flyers (the remaining 15%) exhibited an outstanding profitability, with a return larger than five times their initial investment.

Notice that the values taken by some parameters of the models suggest if a project resembles one of these three profiles. In particular, for a given pair of innovation (a, p), a high flyer should exhibit a priori a larger k, and a poor project, a larger m. In turn, a living dead should a priori show a smaller γ, since this term parametrizes positively (negatively) the probability of extreme (moderate) returns. Accordingly, we will say that γ measures the project's *level of probability extremism.*

Technological shocks. The way how we model the return distribution also allows us to examine the consequences of a change in the profitability of investments caused by real or perceived exogenous technological shocks. For instance, whereas an increase of k can be associated to a technological bubble —like the Internet boom of the beginnings of the century—, an increase of m can be associated to the subsequent burst of such a bubble. Also, as episodes of boom and burst are accompanied by an increase of volatility, we can study this class of effects through the parameter σ, which we will say measures the project's *level of return extremism.*

Innovation and risk. Notice that restrictions on parameters guarantee that $\partial\pi_i^{(a,p)}/\partial a > 0$ and $\partial\pi_i^{(a,p)}/\partial p > 0$ for $i = 1$ and $i = 3$, but $\partial\pi_i^{(a,p)}/\partial a < 0$ and $\partial\pi_i^{(a,p)}/\partial p < 0$ for $i = 2$. Thus, a higher level of either a or p increases (decreases) the probability of extreme (moderate) events. This property is consistent with the idea that innovation parametrize the project's risk, and with the fact that real-world high-tech projects face high degrees of uncertainty, either technological, business/market-related or regulatory, especially in their early stages (Metric and Yasuda, 2010).

In statistical terms, this characteristic implies that the so-called *monotone likelihood ratio property* (MLRP) —a classic condition in contract theory— is not satisfied. To illustrate this phenomenon, let us define the likelihood ratio associated to innovation a as

$$LR_i^a = \frac{\partial\pi_i^{(a,p)}/\partial a}{\pi_i^{(a,p)}}, \text{ for } i = 1, 2, 3,$$

from which it is easy to verify that $LR_1^a > LR_2^a$ as

$$LR_2^a = \frac{1}{e - (\gamma(1+m))^{-1}} < 0 < \frac{1}{e} = LR_1^a, \tag{3.1}$$

despite $x_2 > x_1$.[8]

Recall that LR_i^a indicates how good is the result x_i as a signal of the fact that the entrepreneur had selected an innovation level a. In other words, LR_i^a reflects how *informative* is the project's return x_i (verifiable) with respect to an innovation decision (unverifiable). Then, the larger LR_i^a, the more likely that the entrepreneur had chosen an innovation level a. Thus, if the investor wants this innovation degree to be selected, the financial contract should compensate more the entrepreneur whenever the cash flow x_i is observed. Nevertheless, note that this fact does not guarantee that the optimal financing scheme must be increasing in the result. The last property only holds if the likelihood ratio is monotonically increasing in the return x_i, that is, if the MLRP is met. Indeed, as it is established later, the no verification of this property in our model is crucial to attain the main results of the present article.

Definition 1. Let us define Ψ, the project's *expected marginal return of innovation*, as

$$\Psi \equiv \frac{\partial E(x_i^{(a,p)})}{\partial a} = \frac{\partial E(x_i^{(a,p)})}{\partial p}.$$

Thus, Ψ represents how much the expected return of the project changes when innovation a or p increases. Specifically, in our model this term is given by

$$\begin{aligned} \Psi &= m\gamma(x_1 - x_2) + \gamma(x_3 - x_2) \\ &= \gamma(\sigma(k - m) + (k - 1)). \end{aligned} \tag{3.2}$$

We impose two bounds over Ψ. From below, we assume that $\Psi > 0$, i.e., that the effect of innovation on a higher expected return conditional on success —the positive term $\gamma(x_3 - x_2)$— exceeds the effect on a lower expected return conditional on failure —the negative term $m\gamma(x_1 - x_2)$. This implies to assuming that an increase of innovation a or p moves the return distribution toward more profitable riskier ventures. From above, we assume that $\Psi < \frac{1}{2}$. This upper bound ensures that the first-best joint innovation level e is strictly smaller than 1, which avoids a corner solution (see Proposition 1).

Given the conditions for γ and m set by assumption (A1), these two bounds over Ψ are guaranteed by the following technical assumptions.

Assumption 2 (A2). (i) The level of abnormal success parameter k lies in the interval

$$\underline{k} \equiv \frac{1 + \sigma m}{1 + \sigma} < k < \frac{2 + \sigma}{1 + \sigma} \equiv \overline{k}.$$

(ii) Parameters σ and m satisfy

$$\sigma(m - 1) < 1.$$

[8] The inequalities in expression (3.1) hold because $e \in [0, 1]$ and $\gamma \in \left(0, \frac{1}{1+m}\right)$ by assumption (A1).

As we will see in next sections, these conditions on parameters will turn out also to be useful when performing numerical simulations.

3.2 Preferences

We assume that the entrepreneur's preferences are described by the following (ex post) additively separable risk-neutral utility function:

$$U(w, a) = w - \frac{a^2}{2},$$

where w represents the entrepreneur's share over the return generated by the project and the second term is his disutility from selecting a positive level of innovation. Whereas the financing contract must satisfy a limited liability constraint on the entrepreneur side, and thus, $w \geq 0$, there is not such constraint on the investor side.

The investor is also risk neutral and her preferences (before initial investment I) are represented by the (ex post) utility function

$$B(x, w, p) = x - w - \frac{p^2}{2},$$

where $x - w$ is the share the investor receives from the project's return and the last term is the disutility she experiences from undertaking innovation.

Since the optimal financing scheme can be contingent on returns, for simplicity we define $w_i \equiv w(x_i)$, the entrepreneur's share if return x_i is observed for all $i = 1, 2, 3$.

The opportunity cost of the investor is the riskless interest rate r, and thus, the optimal contract must satisfy that her gross expected payoff be greater or equal than $(1+r)I$, which however becomes zero because the normalization of I. All the bargaining power is in the investor's hands, and thus, the optimal contract must offer to the entrepreneur at least his reservation utility. This utility is represented by $\underline{U}$, which must satisfy the following assumption.

Assumption 3 (A3). The entrepreneur's reservation utility is such that

$$\frac{3}{8}\Psi^2 < \underline{U} < 1 + \frac{3}{4}\Psi^2.$$

To setting these two bounds over $\underline{U}$ is not just a simplifying assumption, since it guarantees two important results in the context of our analysis. First, the lower bound ensures that, under asymmetric information, the optimal w_2 will be strictly positive (see Proposition 2), which rules out that a debt-like scheme may be one of the optimal arrangements (see Corollary 1, case (c1)). Second, the upper bound of $\underline{U}$ ensures that, under symmetric and asymmetric information, the investor's participation constraint is satisfied, and thus, it prevents the existence of a credit rationing equilibrium.

Finally, in order to compare the optimal schemes under symmetric and asymmetric information, we define the investor's optimal expected payoff as[9]

$$EB^j = \sum_{i=1}^{3} \pi_i^{\left(a^j, p^j\right)} \left(x_i - w_i^j\right) - \frac{(p^j)^2}{2} \tag{3.3}$$

where j indicates the type of sharing rule: first-best ($j = FB$) and second-best contract ($j = *$). Similarly, we define the expected surplus generated by the sharing rule j as

$$S^j = \sum_{i=1}^{3} \pi_i^{\left(a^j, p^j\right)} x_i - \frac{(a^j)^2}{2} - \frac{(p^j)^2}{2}. \tag{3.4}$$

4 Results

In this section we characterize the optimal innovation pair and its associated financing rule under both symmetric and asymmetric information. Then, we center our analysis on sharing rules that reward risk-taking and, hence, failure.

4.1 First-Best Solution

Under symmetric information, innovation decisions a and p are contractible, and thus, the optimal sharing rule must solve the following program:[10]

$$\underset{\{w_i\}_{i=1}^3,\ a,p\in[0,1]}{Max} \sum_{i=1}^{3} \pi_i^{(a,p)} \left(x_i - w_i\right) - \frac{p^2}{2} \tag{4.1}$$

$$s.t.$$

$$\sum_{i=1}^{3} \pi_i^{(a,p)} w_i - \frac{a^2}{2} \geq \underline{U} \tag{4.2}$$

$$\sum_{i=1}^{3} \pi_i^{(a,p)} \left(x_i - w_i\right) - \frac{p^2}{2} \geq 0 \tag{4.3}$$

$$w_i \geq 0 \text{ for all } i \tag{4.4}$$

$$a + p \leq 1, \tag{4.5}$$

where (4.2) and (4.4) are the participation constraint and the limited liability constraint for the entrepreneur, respectively; (4.3) is the participation constraint for the investor; and (4.5) is the constraint that ensures that the joint innovation is not greater than 1.

[9] Because we normalized I to zero, this is indeed an expected payoff before and after initial investment. The same applies to the expected surplus defined below.

[10] Because we normalized initial investment to zero, we exclude I to compute the investor's expected payoff. However, notice that because I is an exogenous variable in our fixed-investment model, even if this term were positive, it would also be excluded from the investor's objective function in all the optimization programs posed in this paper.

After solving this program, we can establish the following result.

Proposition 1. *Under assumption (A1)-(A3), the first-best incentive scheme is such that*

$$\begin{aligned} a^{FB} &= \Psi \\ p^{FB} &= \Psi \\ e^{FB} &= 2\Psi, \end{aligned}$$

and it can be implemented by two types of sharing rules:
(i) A full-insurance contract for the entrepreneur that involves a fixed compensation

$$w^{FB} = \underline{U} + \frac{\Psi^2}{2}$$

if $a = a^{FB}$*, and a penalization otherwise.*
(ii) A full-franchise contract that involves a fixed payment from the entrepreneur to the investor given by[11]

$$F^{FB} = 1 + \frac{3\Psi^2}{2} - \underline{U}$$

if $p = p^{FB}$*, and a penalization otherwise.*

Proof. See the Appendix.

Thus, under symmetric information, there is a *perfect balanced* solution in terms of innovation, as both a and p are positive and identical. Whereas the existence of interior solutions comes from assumption (A2), the symmetric solution is a consequence of assuming that the investor's and the entrepreneur's innovation are equally efficient in terms of the social value of the project. In fact, we assumed that both innovations are perfect substitutes and share the same cost function. As a result, the social marginal expected benefit of a and p are identical, as well as their social marginal cost, which finally explains the symmetry of their optimal levels.

[11] The upper bound imposed by assumption (A3) on the entrepreneur's reservation utility implies that F^{FB} is always positive.

4.2 Second-Best Solution

Under asymmetric information, the optimal sharing rule must solve the following program:

$$\underset{\{w_i\}_{i=1}^3,\ a,p\in[0,1]}{Max} \sum_{i=1}^{3} \pi_i^{(a,p)} (x_i - w_i) - \frac{p^2}{2} \tag{4.6}$$

$$s.t.$$

$$\sum_{i=1}^{3} \pi_i^{(a,p)} w_i - \frac{a^2}{2} \geq \underline{U} \tag{4.7}$$

$$\sum_{i=1}^{3} \pi_i^{(a,p)} (x_i - w_i) - \frac{p^2}{2} \geq 0 \tag{4.8}$$

$$a \in \arg \max_{\tilde{a}\in[0,1]} \sum_{i=1}^{3} \pi_i^{(\tilde{a},p)} w_i - \frac{\tilde{a}^2}{2} \tag{4.9}$$

$$p \in \arg \max_{\tilde{p}\in[0,1]} \sum_{i=1}^{3} \pi_i^{(a,\tilde{p})} (x_i - w_i) - \frac{\tilde{p}^2}{2} \tag{4.10}$$

$$w_i \geq 0 \text{ for all } i \tag{4.11}$$

$$a + p \leq 1, \tag{4.12}$$

where (4.7), (4.9), and (4.11) represent the participation constraint, the incentive compatibility constraint and the limited liability constraint for the entrepreneur, respectively; and (4.8) and (4.10) is the participation constraint and the incentive compatibility constraint for the investor, respectively.

After solving this program, we can establish the following result.

Proposition 2. *Under assumptions (A1)-(A3), the second-best incentive scheme is such that the optimal innovation levels are:*

$$a^* = \frac{\Psi}{2}$$

$$p^* = \frac{\Psi}{2}$$

$$e^* = \Psi,$$

and it can be implemented by a sharing rule characterized by the following set:

$$\Omega^* = \left\{ (w_1^*, w_2^*, w_3^*) : w_1^* \in \left[0, \frac{A}{m}\right], w_2^* = \underline{U} - \frac{3}{8}\Psi^2, w_3^* \in [0, A] \text{ and } w_3^* = A - mw_1^* \right\},$$

with

$$A \equiv \frac{\Psi}{2\gamma} + (1+m)(\underline{U} - \frac{3}{8}\Psi^2).$$

Proof. See the Appendix.

In terms of innovation, the optimal solution under asymmetric information is, like under symmetric information, perfectly balanced. As previously discussed, this result is the consequence of assuming a symmetric environment regarding the social marginal benefits and costs of both a and p. The optimal values are, however, smaller than those under full information —in fact the fifty percent— which implies that despite risk-neutrality, there is no financial contract that can achieve the efficient joint innovation level.

In terms of financial contracts, the set Ω^* implies that there are multiple sharing rules that implement the second-best innovation levels. This multiplicity of contracts is illustrated in Fig. 1, in which becomes clear that the optimal contract allows a wide range of financing rules that includes strictly increasing, nonlinear, nonmonotone, and even strictly decreasing schemes. This multiplicity of contracts is characterized in the next statement.

⟨Insert Fig. 1 here⟩

Corollary 1. *The second-best financing rule is such that all these classes of contracts are optimal:*

(a) Strictly increasing contracts: $w_3^* > w_2^* > w_1^*$.

(b) Strictly decreasing contracts: $w_1^* > w_2^* > w_3^*$.

(c) Non-monotone contracts:

- *(c1) Bonus plus fixed compensation:* $w_3^* > w_2^* = w_1^*$.
- *(c2) J-shaped contract:* $w_3^* > w_1^* > w_2^*$.
- *(c3) U-shaped contract:* $w_3^* = w_1^* > w_2^*$.
- *(c4) G-shaped contract:* $w_1^* > w_3^* > w_2^*$.
- *(c5) L-shaped contract:* $w_1^* > w_3^* = w_2^*$.

Proof. It follows directly from Proposition 2 and Fig. 1.

Thus, when MLRP is not verified what indeed measures the *power of incentives* of the optimal contract is not its *shape* or its *average slope*, but a given relationship of rewards paid to the extreme returns. In fact, the negative relationship between w_3^* and w_1^* described by equation $w_3^* = A - mw_1^*$ in Proposition 2 (and illustrated in Fig. 1) means that it is irrelevant whether the average slope of the entire sharing rule increases or decreases (even if it changes from positive to negative). All this makes sense because, according assumption (A1), whereas extreme returns (either high or low) are more indicative of a higher level of innovation, the moderate return is indicative of a lower innovation level. A consequence of the same phenomenon is that, for instance, a scheme that rewards moderate returns (an inverse-U-shaped contract) is excluded from the wide range of optimal financing rules characterized in Corollary 1.

Two schemes of this wide family of optimal contracts deserve a more-in-depth comment: the strictly decreasing and the risk-reward scheme. In the particular case of the counter-intuitive strictly decreasing contract, its optimality can be explained, in addition to the absence of the MLRP, on the basis of the double moral hazard environment. In that setting, incentives are provided by the combination of the reward/punishment scheme applied to the agent *and* the principal, as both of them contribute with innovation to an entrepreneurial business venture. Thus, although a decreasing contract $w(x)$, in relative terms, rewards the entrepreneur for failure and punishes him for success, the complementary optimal scheme $x - w(x)$ provides the investor with the opposite incentives. As as consequence, it is finally the interplay of both schemes which induces the optimal individual and joint innovation levels. Under the specific formulation of the model, it is the optimal combination of incentives over the two parties which induces a perfectly balanced solution in terms of innovation.[12]

4.3 Risk-reward contracts

The second class of contracts that deserves a more detailed analysis is that concerning risk-reward schemes, i.e., those contracts that, in relative terms, reward extreme returns and punish moderate results. Notice that in this class of financing rules $w_1^* > w_2^*$, and hence, they promise a larger share to the entrepreneur when the observed return is low (x_1) than when it is intermediate (x_2). Thus, we cannot rule out that a scheme with a *reward-for-failure* property be an optimal arrangement when financing highly innovative ventures. Examples of this class of schemes are the J-shaped and U-shaped contracts identified in Corollary 1.

Despite these contracts are counter-intuitive, they have, in the context of entrepreneurial business financing, an appealing economic rationale. In practice, it is usual that more innovation brings together more expected returns but also more risk and higher failure rates. Thus, it is reasonable to expect that sometimes (ex post) low returns are a better signal of the selection of a more innovative project than (ex post) moderate returns. Hence, it may be optimal for the investor to reward the entrepreneur when extreme returns are observed (either sufficiently high or sufficiently low), and to punish him (in relative terms) when intermediate returns take place. Therefore, it may be efficient to reward for failure, as long as low returns be sufficiently suggestive of high effort and creativity on the part of the entrepreneur to propose investment projects

[12] Wang (2004) also characterizes the optimal financial contracts under double moral hazard, but assuming MLRP and risk-aversion for the agent. In his setup, one of the optimal contracts exhibits a lesser severe nonmonotonicity than our strictly decreasing scheme, which is, however, explained by the absence of a limited liability condition on the principal side.

with a better profile of risk and expected return.[13]

Our analysis suggests that this reward-for-failure property may explain some non-linear and non-monotone characteristics of schemes usually adopted to fund highly innovative firms. This issue is explored in Section 5, which focus on the implementation of the optimal sharing rule via real-world financing structures that mix debt, equity and conversion clauses.

4.4 First-best vs. second best contracts

We end this section by summarizing our main results in terms of innovation levels, investor's expected payoff and expected surplus. We also compare the optimal schemes under symmetric and asymmetric information in terms of these three dimensions as follows.

Corollary 2. *(i) The innovation levels under symmetric information are strictly greater than those induced under asymmetric information:*

$$\begin{aligned} a^{FB} &= \Psi > \frac{\Psi}{2} = a^* \\ p^{FB} &= \Psi > \frac{\Psi}{2} = p^* \\ e^{FB} &= 2\Psi > \Psi = e^*. \end{aligned}$$

(ii) The investor's expected payoff under symmetric information is strictly greater than that under asymmetric information:

$$EB^{FB} = 1 + \Psi^2 - \underline{U} > 1 + \frac{3}{4}\Psi^2 - \underline{U} = EB^*.$$

(iii) The expected surplus generated under symmetric information is strictly greater than that generated under an asymmetric-information setup:

$$S^{FB} = EB^{FB} + \underline{U} > EB^* + \underline{U} = S^*.$$

Proof. See the Appendix.

It is well known that, under a setup with double moral hazard and MLRP, there is no incentive scheme that induces the first-best effort level even when the agent is risk neutral (Holmstrom, 1982). Thus, the present work extends such a result to a model with double moral hazard and *no* MLRP, as established in point (i) of Corollary 2.

[13] A similar idea of "reward for failure" is also discussed by Manso (2011) in the context of exploration of technological innovation, and Loyola and Portilla (2014) in the context of top executive compensation.

5 Comparative statics

This section presents the results of two classes of exercises: (i) an analysis of comparative statics with respect to different parameters of the return distribution; and (ii) a comparative statics exercise on an optimal scheme of the risk-reward type.

5.1 Comparative statics on optimal contracts

We perform a comparative statics analysis concerning four parameters of the return distribution, whose results are summarized as follows.

Corollary 3. *The first-best and second-best innovation and surplus levels are:*
(i) Increasing in γ*.*
(ii) Increasing in k*.*
(iii) Decreasing in m*.*
(iv) Increasing (decreasing) in σ *as long as* $k > m$ *(*$k < m$*).*

Proof. See the Appendix.

Our model allows then us to examine how innovation depends on the primitives of the projects, that is, the parameters of the return space and of the return distribution.

An application of this feature of our model concerns an analysis of the effects of technological shocks. For instance, results of Corollary 3 indicate that when the return distribution becomes more positively skewed, the sign of the final effect on optimal levels of innovation and surplus will depend on the source of this larger skew. If the source is an increase in the either real or perceived success return (an increase of k), as for instance that generated by a positive technological change, this will finally have a positive effect on the social value of the project. An example of this type of episode would be the Internet bubble of the beginning of this century. On the other hand, if the source of a larger skewness on the return distribution is an increase in the real or perceived failure rates of projects (an increase of m), as for instance that provoked by the proceeding burst of the Internet bubble, this will finally have a negative effect on the social value created by the project.

In addition, our framework also allows us to predict the effects of a larger volatility in the returns of highly innovative projects provoked, for instance, by a bubble-burst dynamic like that discussed previously. In this vein, corollary 3 suggests that the effect of a higher degree of extremism, represented by an increase of parameter σ, will depend on the phase of the cycle. During a boom, our model predicts that an increase of σ will have a positive effect on innovation levels and the social value of projects, as in an upward phase it is more likely that $k > m$. By contrast, during the burst of a bubble, our model predicts that an increase of σ will have a negative effect on innovation and its social value, as in a downward phase it is more likely the other way around with k and m.

We also establish the next results about the *gap* between the first-best and second-best contracts, which can be thought of as a measure of the cost of asymmetric information.

Corollary 4. *The gap between first-best and second-best innovation levels and the gap between first-best and second-best expected surplus are:*
(i) Increasing in γ.
(ii) Increasing in k.
(iii) Decreasing in m.
(iv) Increasing (decreasing) in σ *as long as* $k > m$ *(*$k < m$*).*

Proof. See the Appendix.

Thus, in the case of highly innovative projects, the welfare cost of asymmetric information will increase with the return skew towards success and will decrease with the probability skew towards failure.

To illustrate the main findings of this subsection, we carry out several numerical simulations considering changes in all the parameters of the return distribution. As examples, we present the results of the exercises conducted for γ, m and k in figures 2-7.[14] In the exercises performed for the two first parameters, we take $\sigma = 0.5$ and $k = 1.6$ so that the space of returns is assumed to be $X = \{0.5, 1, 2.4\}$. In the specific case of the comparative statics analysis for the level of probability extremism, we suppose in addition that $m = 1.5$ and hence, $\gamma \in (0, 0.4)$. In turn, in the specific case of the probability skew towards failure, we suppose in addition that $\gamma = 0.35$, and hence, $m \in (1, 1.8571)$.

For the numerical exercises conducted over the return skew towards success, we assume that $\sigma = 0.5$, $m = 1.5$ and $\gamma = 0.35$. Thus, we consider the space of returns $X = \{0.5, 1, 1.5k\}$ such that $k \in (1.17, 1.67)$.[15]

The results of these simulations are displayed in figures 2-7, all of which confirm the comparative statics properties established in corollaries 2-4. Moreover, these simulations also allow us to predict the performance of a project belonging to some of the three empirical classes described in Subsection 2.1, namely, poor projects, living dead and high flyers.

Living dead. Numerical simulations of figures 2 and 3 suggest that a project with a profile closer to a living dead (i.e., with a low value of γ) should, in fact, exhibit

[14] The results of numerical exercises for σ are also available upon the authors' request.

[15] It is possible to check that the value constellations considered in the numerical simulations concerning all the parameters satisfy assumptions (A1) and (A2).

lower optimal innovation levels and a *lower* social value for its innovation. The welfare cost of asymmetric information should be, however, *lower* for this class of projects.
⟨Insert Fig. 2 and 3 here⟩

Poor project. Numerical exercises displayed in figures 4 and 5 reveal that a venture with a profile, a priori, closer to a poor project (i.e., with a high value of m) should, in fact, yield *lower* levels of both optimal innovation and social value of this innovation. Nevertheless, this type of projects should suffer a *lower* social lost due to asymmetric information.
⟨Insert Fig. 4 and 5 here⟩

High flyer. Numerical simulations of figures 6 and 7 indicate that a project with a profile closer to a high flyer (i.e., with a large value of k) should, in fact, exhibit a *higher* level of optimal innovation and a *higher* social value created by this innovation. However, the welfare costs attributable to asymmetric information should also be *larger* for this class of projects.
⟨Insert Fig. 6 and 7 here⟩

5.2 Comparative statics on a risk-reward scheme

We also perform a comparative statics exercise on the U-shaped contract, an optimal financing rule of the risk-reward type. Since in this particular contract, $w_3^* = w_1^* = w^* > w_2^*$, we can define Δw^*, the *optimal reward-for-risk,* as follows

$$\Delta w^* \equiv w^* - w_2^*,$$

such that Δw^* represents a reward for either success or failure. This reward-for-risk measures the power of incentives the U-shaped contract gives to the entrepreneur to undertake innovation. It is possible to show that

$$\Delta w^* = \frac{\sigma(k-m) + (k-1)}{2\,(1+m)},$$

from which it is simple to check that

$$\begin{aligned} \frac{\partial \Delta w^*}{\partial m} &= -\frac{\sigma(k+1)+k-1}{2\,(m+1)^2} < 0, \\ \frac{\partial \Delta w^*}{\partial k} &= \frac{\sigma+1}{2m+2} > 0, \end{aligned}$$

and

$$\frac{\partial \Delta w^*}{\partial \sigma} = \frac{k-m}{2m+2} > 0,$$

if $k > m$.

Notice that the sign of these partial derivatives is equal to that established in Corollary 3 by the comparative statics analysis on the innovation and surplus levels of the optimal contracts. As a result, the effects of changes in the parameters m, k, and σ on the optimal reward-for-risk Δw^* can be interpreted quite intuitively. In fact, these results imply that the power of incentives of a U-shaped financing contract over the entrepreneur will be higher whenever exogenous changes on either the return space or the return distribution generate higher levels of optimal innovation.

6 Implementation

We perform two implementation analysis, one regarding optimal financing schemes and the other concerned with suboptimal schemes.

6.1 Optimal financing rules

We analyze the implementation of some optimal financing rules characterized in Corollary 1 through three classes of hybrid securities: (i) convertible preferred equity with optimal dividend, (ii) a sequence of warrants, and (iii) an initial equity stake with either short positions in call options or dilution in favor of the investor.

All these examples highlight the idea that the optimal financing rule must contain some level of preferred equity for the entrepreneur and some level of common equity for the investor. In a double moral hazard setup, a proper combination of these two claims delivers sufficiently high-powered incentives to induce both parties to undertake a positive level of innovation, and under our specific symmetric environment, to achieve a perfectly balanced solution.

6.1.1 Convertible preferred equity

To study this class of securities, we focus on the particular optimal strictly increasing contract $w_3^* > w_2^* > w_1^*$ in which[16]

$$w_1^* = 0, \tag{6.1}$$

$$w_2^* = \underline{U} - \frac{3}{8}\Psi^2, \tag{6.2}$$

$$w_3^* = A. \tag{6.3}$$

Following Marx (1998), we consider a financing contract in which the investor receives an ex post payoff described by

$$x - w(x) = \begin{cases} x & \text{if } x < d \\ d + \beta(x-d) & \text{if } x \geq d \end{cases},$$

[16] This contract follows from substituting $w_1^* = 0$ into Proposition 2.

where $d > 0$ represents a fixed dividend payment. Thus, this contract can be interpreted as a convertible preferred stock, whose holder has seniority over a preferred dividend d and converts this preferred equity in a common one —at a rate β— once the project's return exceeds the threshold d.

In the context of our model, we must then look for the optimal pair (d^*, β^*) that awolls us to implement the contract (6.1)-(6.3), so that it satisfies

$$(1-\beta^*)(x_2 - d) = w_2^*, \quad (6.4)$$

$$(1-\beta^*)(x_3 - d) = w_3^*, \quad (6.5)$$

as long as $d \in (x_1, x_2)$. After combining equations (6.2), (6.3), (6.4) and (6.5), and substituting x_2 and x_3, it yields

$$d^* = \frac{\frac{\Psi}{2\gamma} + (1 + m - k(1+\sigma))(\underline{U} - \frac{3}{8}\Psi^2)}{\frac{\Psi}{2\gamma} + m(\underline{U} - \frac{3}{8}\Psi^2)}, \quad (6.6)$$

and

$$\beta^* = 1 - \frac{\underline{U} - \frac{3}{8}\Psi^2}{1 - d^*}. \quad (6.7)$$

From (6.6), it can be shown that whereas indeed $d^* < x_2$ (as $k(1+\sigma) > 1$), it is true that $d^* > x_1$ only if

$$\frac{k(1+\sigma) - (1-\sigma)}{\sigma} < 1 + \frac{\sigma(k-m) + k - 1}{2(\underline{U} - \frac{3}{8}\Psi^2)} + m. \quad (6.8)$$

To illustrate this type of implementation, we simulate results taking the parameter values $\gamma = 0.35$, $\sigma = 0.5$, $k = 1.6$, $m = 1.5$, and $\underline{U} = 0.15$, which generates the return space $X = \{0.5, 1, 2.4\}$. The optimal financing rule is then given by $w_1^* = 0$, $w_2^* = 0.130\,59$, and $w_3^* = 0.651\,48$, which can be implemented by a preferred equity with dividend $d^* = 0.598\,86$ and a conversion rate equal to $\beta^* = 0.674\,45$. Thus, the investor's payoff profile under this convertible security is given by

$$x - w^* = \begin{cases} x & \text{if } x < 0.598\,86 \\ 0.598\,86 + 0.674\,45(x - 0.598\,86) & \text{if } x \geq 0.598\,86 \end{cases},$$

which is displayed in Fig. 8.

⟨Insert Fig. 8 here⟩

6.1.2 Sequence of warrants

As in the case of the convertible preferred stock, we center our analysis on the same optimal strictly increasing contract represented by equations (6.1)-(6.3).

Inspired by the analysis of Repullo and Suarez (2004), we now consider a financing contract that includes an investor's initial outside equity and various thresholds for the project's return from which the investor's share changes. This contract can be viewed as preferred equity plus a set of warrants that yields higher payoffs to the investor as long as the firm value hits higher and higher strike prices.[17]

In more concrete terms, first the preferred equity entails the investor with a senority over a dividend $d = d_1$, and second, given two strike prices (d_1, d_2) such that $d_1 < d_2$, the investor also holds long position in two call options over additional equity. Then, if the project's return $x \in (d_1, d_2]$, the investor has the option to buy an additional stake of equity β_1 to a unitary exercise price equal to d_1. Alternatively, if the return is sufficiently high so that $x > d_2$, the investor can then buy an extra equity stake β_2 to a unitary exercise price of d_2.

Thus, the investor's ex post payoff is represented by the structure[18]

$$x - w(x) = \begin{cases} x & \text{if } x \leq d_1 \\ d_1 + \beta_1 \max\{x - d_1, 0\} & \text{if } d_1 < x \leq d_2 \\ d_1 + \beta_2 \max\{x - d_2, 0\} & \text{if } x > d_2 \end{cases}.$$

In particular, if we fix exogenously strikes prices to be $d_1 = x_1$ and $d_2 = x_2$, we need to find out what is the optimal pair (β_1^*, β_2^*) that implements the contract described by (6.1)-(6.3), so that it has to satisfy

$$(1 - \beta_1^*)(x_2 - x_1) = w_2^*, \tag{6.9}$$

$$(x_3 - x_1) + \beta_2^*(x_2 - x_3) = w_3^*, \tag{6.10}$$

After combining equations (6.2), (6.3), (6.9) and (6.10), and replacing values of x_i's, we obtain

$$\beta_1^* = 1 - \frac{\underline{U} - \frac{3}{8}\Psi^2}{\sigma},$$

$$\beta_2^* = 1 + \frac{A - \sigma}{1 - k(1 + \sigma)}.$$

We illustrate this implementation with a numerical example by taking the same parameter values used for the case of the convertible preferred equity. We then get $\beta_1^* = 0.73882$ and $\beta_2^* = 0.8918$, and hence, the investor's payoff profile under this warrant-based structure is

$$x - w(x) = \begin{cases} x & \text{if } x \leq 0.5 \\ 0.5 + 0.738\,82(x - 0.5) & \text{if } 0.5 < x \leq 1 \\ 0.5 + 0.891\,8(x - 1) & \text{if } x > 1 \end{cases}$$

which is shown in Fig. 9.

⟨Insert Fig. 9 here⟩

[17] Since our framework is a one-stage financing setup, this contract can also be interpreted as a convertible bond with a coupon equal to the preferred dividend.

[18] Notice that, conditional on one of these two warrants being active, the call option involved in each of these contracts will always be "in the money".

6.1.3 Equity, call options, and dilution

Finally, we analyze a mixed financing structure that combines outside/inside equity, and either short positions of the investor in call options or an equity dilution process. This hybrid structure enables the investor to potentially infuse new funds in case of low returns are observed, allowing thus a cross-subsidization between incomes coming from good and bad states of nature.

To this end, we examine how to implement the class of optimal risk-reward schemes analyzed in Subsection 3.3. For simplicity, we focus on the U-shaped contract in which $w_1^* = w_3^* = w^*$, and whose specific sharing rule —obtained from Proposition 2— becomes

$$w^* = \frac{A}{1+m}, \tag{6.11}$$

and

$$w_2^* = \underline{U} - \frac{3}{8}\Psi^2. \tag{6.12}$$

For simplicity, we additionally suppose that $x_1 > 0$ and normalize the number of initial shares to 1. Two cases then arise depending on the possibility of infusing new funds on the part of the investor.

Case 1: $w^* > x_1$. In this case, the U-shaped contract can be implemented by a combination of a full-outside equity stake for the investor and a set of call options over different equity stakes depending on the project's returns. A detailed description of this mixed security structure is characterized as follows.

Corollary 5 (Case 1). *Assume that $w^* > x_1$. The U-shaped contract can be implemented by giving the investor an initial full equity stake and giving the investor (entrepreneur) a short (long) position in a set of equity call options triggered according to the following scheme:*
(i) If $x \leq x_1$, a call option over an equity stake $(1+\delta_1)$ with a zero strike price is active. The exercise of this option requires previously both the investor to infuse new funds by an amount $\delta_1 x_1$ and the issue of an amount $\delta_1 > 0$ of new shares such that

$$\delta_1 = \frac{A}{(1+m)(1-\sigma)} - 1.$$

(ii) If $x_1 < x \leq x_2$, a call option over an equity stake $(1+\delta_2)$ with a zero strike price is active, where $\delta_2 > -1$ is such that

$$\delta_2 = \underline{U} - \frac{3}{8}\Psi^2 - 1.$$

(iii) If $x > x_2$, a call option over an equity stake $(1+\delta_3)$ with a zero strike price is

active, where $\delta_3 > -1$ *is such that*

$$\delta_3 = \frac{(1+\delta_1)(1-\sigma)}{k(1+\sigma)} - 1.$$

Proof. See the Appendix.

This implies that the investor's ex post payoff is represented by the following structure:

$$x - w(x) = \begin{cases} x - (1+\delta_1)\max\{x,0\} & \text{if } x \leq x_1 \\ x - (1+\delta_2)\max\{x,0\} & \text{if } x_1 < x \leq x_2 \\ x - (1+\delta_3)\max\{x,0\} & \text{if } x > x_2 \end{cases}, \quad (6.13)$$

which reflects the difference between the full-initial-equity payoff and the losses coming from the short position held by the investor in each call option.

Therefore, if the lowest return of the project x_1 is sufficiently low (i.e., $x_1 < w^*$), this mixed financing structure considers the infusion of new resources from the investor when the worst state of nature occurs. This result is the consequence of the reward-for-failure feature of the optimal risk-reward scheme. Notice, however, that since the investor's expected payoff at the optimal contract must be positive (as at the optimal solution the investor's participation constraint is satisfied with inequality), at least one of the ex post payoffs of the other states of nature must be positive. From expression (6.13), this implies that either δ_2 or δ_3 (or both) must be strictly negative, which ensures that the investor enjoys a stake over the success returns of the project, and thus, a combination of outside and inside equity must be adopted.

Overall, from this analysis we can then conclude that risk-reward sharing rules allow the investor to *cross-subsidize* incomes of good and bad states of nature. (**referenciar paper anterior**)

A numerical illustration of this phenomenon is performed assuming $\sigma = 0.5$, $k = 1.6$, $m = 1.5$, $\underline{U} = 0.6$, and $\gamma = 0.35$, which generates a return space $X = \{0.5, 1, 2.4\}$. The optimal financing rule is then given by $w^* = 0.7106$ and $w_2^* = 0.58059$, which can be implemented by the mixed security structure of Corollary 5 with the equity stakes $\delta_1^* = 0.421\,2$, $-\delta_2^* = 0.419\,41$, and $-\delta_3^* = 0.70392$. Hence, the investor's payoff profile under this scheme becomes[19]

$$x - w(x) = \begin{cases} x - 1.421\,2\max\{x,0\} & \text{if } x \leq 0.5 \\ 0.419\,41x & \text{if } 0.5 < x \leq 1 \\ 0.703\,92x & \text{if } x > 1 \end{cases},$$

which is displayed in Fig. 10.

⟨Insert Fig. 10 here⟩

[19] In this numerical example the two last call options are always "in the money".

Case 2: $w^* \leq x_1$. In this case, the U-shaped contract can be implemented by an initial profit-sharing rule that gives to the investor an equity stake of $(1-\alpha)$ over the project's return, and hence, an initial combination of outside and inside equity is optimal. It also considers the posterior issue of an additional amount of shares ϕ_i in favor of the investor when a return x_i $(i = 2, 3)$ is observed. This additional equity thus dilutes the participation of the entrepreneur to a fraction $\frac{\alpha}{1+\phi_i}$ of the project's outcome. The specific values of these stakes and shares are characterized as follows.

Corollary 6 (Case 2). *Assume that* $w^* \leq x_1$*. A U-shaped financing rule can be implemented by giving an initial equity stake* $(1-\alpha) \in [0, 1)$ *to the investor such that*

$$\alpha = \frac{A}{(1+m)(1-\sigma)},$$

and by issuing new shares in her favor according to the following scheme:
(i) If $x_i = x_2$*, this additional amount of investor's shares is:*

$$\phi_2 = \frac{\alpha - \underline{U} + \frac{3}{8}\Psi^2}{\underline{U} - \frac{3}{8}\Psi^2}.$$

(ii) If $x_i = x_3$*, this additional amount of investor's shares is:*

$$\phi_3 = \frac{k(1+\sigma) + \sigma - 1}{1-\sigma}.$$

Proof. See the Appendix.

6.2 Suboptimal financing rules

We examine two interesting suboptimal financing schemes: the straight debt and the full-franchise contracts.

6.2.1 Debt contract

The pure debt is a particularly interesting scheme since it represents the sharing rule involved in traditional sources of financing such as a bank loan or a bond issue. In contrast to the framework here adopted, these conventional financial contracts in general involve an environment of *single* moral hazard, as neither the bank nor bondholders provide advise or managerial effort to the entrepreneur in the process of selecting and/or managing innovative investment projects. Thus, in order to illustrate how much optimal hybrid securities perform better than a straight debt contract under a double moral hazard setup, we conduct some numerical comparative exercises.

To this end, we study a debt scheme with a coupon $D = x_1$ such that the investor's ex post payoff (before disutility of innovation) becomes[20]

$$x - w(x) = \begin{cases} x & \text{if } x \leq x_1 \\ x_1 & \text{if } x > x_1 \end{cases}.$$

In the context of our particular three-outcome model, this implies that $w_1^d = 0$, $w_2^d = \sigma$, and $w_3^d = k(1+\sigma) - (1-\sigma)$. After substituting this scheme into the program posed in Subsection 3.2, we obtain that innovation levels that satisfy the incentive compatibility constraints are $a^d = \Psi$, $p^d = 0$ and $e^d = \Psi$. This solution delivers an investor's expected payoff and an expected surplus equal to

$$EB^d = 1 - \sigma, \tag{6.14}$$

and

$$S^d = 1 + \frac{\Psi^2}{2}, \tag{6.15}$$

respectively. This debt contract is, however, admissible if in addition the two participation constraints are satisfied. First, the debt scheme must satisfy the entrepreneur's participation constraint (4.7), which is equivalent to the condition

$$\sigma \geq \underline{U} - \frac{\Psi^2}{2}. \tag{6.16}$$

Second, the debt contract must also satisfy the investor's participation constraint, which is equivalent to the condition

$$\sigma \leq 1. \tag{6.17}$$

Therefore, if either condition (6.16) or (6.17) is not verified, there is no financing through debt, and thus, it emerges an equilibrium that involves a *credit rationing.*

In addition, notice that from Corollary 2 and equation (6.15), it follows that the straight debt scheme cannot implement the second-best solution since

$$S^* > S^d.$$

This result holds because although the debt scheme induces a level of joint innovation equal to that of the second-best contract (i.e., $e^d = e^* = \Psi$), it involves a *corner* solution different from the perfectly balanced solution achieved by the optimal contract. Thus, unlike the optimal solution, a pure debt scheme cannot properly tackle the double moral hazard problem. The reason of this suboptimality relies on the fact that a straight

[20]We also explored the pure debt contract with a coupon $D = x_2$, but our results indicate that it performs much worse than the contract here analyzed.

debt scheme does not provide the same high-powered incentives to the principal and the agent to undertaking innovation. This occurs because a debt scheme generates different return profiles to the claims held by the entrepreneur (inside common equity) and the investor (straight debt). In fact, whereas the contingent and variable return profile of the equity highly motivates the entrepreneur to undertake innovation, the promise of a riskless and fixed repayment of the debt fails to provide the investor with the same level of incentives. By contrast, since our optimal hybrid financial structure always considers some combination of inside and outside equity (see the three structures studied in Subsection 5.1), it is able to provide both sides of the contract with the proper level of incentives.[21]

The foregoing theoretical result is consistent with three stylized facts about the financing schemes used in the most innovative industries. First, hybrid financial structures combining debt and equity, rather than pure debt, are common financing arrangements in high-tech, high-growth, young firms (Kaplan et al., 2012; Sahlman, 1990; Trester, 1998). Second, in this class of financial contracts the investor frequently also provides advise, management or active monitoring to the entrepreneur (Bengtsson and Sensoy, 2011; Kaplan and Stromberg, 2004; Sapienza et al., 1996), which as commented above, does not characterize a conventional debt scheme. Third, the suboptimality of straight debt theoretically established here is also consistent with empirical evidence suggesting that the pecking order theory seems, counter-intuitively, not to be verified in firms with large information asymmetries. This conclusion follows from evidence showing that leverage is lower in high-tech industries than in their low-tech counterparts (Chang and Song, 2014; Frank and Goyal, 2003; Barclay et al., 1995) and that it decreases with R&D expenditures (Long and Malitz, 1985).

To illustrate the suboptimality of the debt scheme, we present the results of numerical simulations about the impact of changes in parameters γ and m over the expected surplus, assuming the same parameters constellation considered in Subsection 4.2.[22] In the case of the level of probability extremism γ, we also fix $\underline{U} = 0.51$, which yields an expected surplus of

$$S^d = 1 + 0.21125\gamma^2,$$

as long as (6.16) is satisfied and thus there is no credit rationing, which is guaranteed by the condition[23]

$$\gamma \geq 0.2176.$$

[21] A result reminiscent of this is established by Casamatta (2003) regarding...**completar.**

[22] The results of simulation exercises concerning other return distribution's parameters are available upon the authors' request.

[23] Since $\sigma = 0.5$, condition (6.17) is satisfied, and thereby, the investor's participation constraint as well.

A graphical comparison between the straight debt scheme and the second-best solution is displayed in Fig. 11, in which we observe two phenomena. First, when using a debt scheme there is a credit rationing for investment projects with a level of probability extremism sufficiently low (the dashed segment of the green curve), or which is equivalent, projects with a mass of probability sufficiently concentrated on moderate returns. In terms of the typology of project discussed in previous sections, this result implies that it is possible that conventional debt structures do not fund highly innovative projects that, despite they yield a positive expected surplus, have an ex ante profile closer to what is called a living dead. In turn, this result highlights that mixed securities are then especially welfare enhancing when financing profitable projects with such characteristics.

Interestingly, this credit rationing result is consistent with empirical evidence showing that small, high-growth, highly innovative firms seem to be more financially constrained (Chang and Song, 2014; Brown, 1997), and that consequently, they use intensively private equity and other equity vehicles rather than banking loans or public bonds.

Second, Fig. 11 also reveals that the gap of expected surplus between the second-best and the straight debt contracts increases with γ. This means that, among the projects that may be funded with debt (the solid segment of the green curve), those with a return distribution more away from a living dead profile would generate a larger increase of social welfare if they instead were funded via an hybrid security structure. ⟨Insert Fig. 11 here⟩

In the case of the probability skew towards failure m, we take the same parameters values and intervals of Subsection 4.2, but now with the additional assumption that $\underline{U} = 0.53$. Hence, the expected surplus from the pure debt contract becomes[24]

$$S^d = 1 + \frac{1}{2}(0.49 - 0.175m)^2,$$

as long as there is no credit rationing, which from (6.16) is equivalent to the condition

$$m \leq 1.4003.$$

Two results emerge form Fig. 12, which compares the performance of the second-best and the straight debt contracts regarding m. First, a debt scheme can involve a credit rationing for projects that generate a positive expected surplus, but exhibit a large probability of failure (the dashed segment of the green curve). This implies that, from a social viewpoint, mixed security structures will do their best when financing profitable

[24] All these parameters values satisfy assumptions (A2) and (A3).

ventures with an ex ante profile close to what is called a poor project, because otherwise this class of projects may not be funded under traditional debt schemes.

Second, we can also observe from Fig. 12 that the gap of expected surplus between the second-best and the straight debt contracts decreases with m. Thus, among the projects that may be financed with debt (the solid segment of the green curve), those with a return distribution with lesser asymmetric tails would generate a larger social welfare improvement if they instead were funded using a mixed security structure.
⟨Insert Fig. 12 here⟩

6.2.2 Full-franchise contract

The franchise contract is an especially interesting case because under *single* moral hazard, this type of scheme induces the first-best effort when the agent is risk neutral as in our model. Previous literature has, however, established that under *double* moral hazard and MLRP, this fixed rent contract is *not* able to implement the first-best solution. We will show that in a double-moral hazard setup with *no* MLRP this result not only holds, but also that the franchise contract is not even a second-best solution.

We then consider the contract

$$w(x) = x - F^f,$$

where F^f is a fixed payment made by the entrepreneur to the investor. The substitution of this contract into the program posed in Subsection 3.2 yields the innovation levels $a^f = \Psi$, $p^f = 0$ and $e^f = \Psi$, and a fixed payment given by

$$F^f = 1 + \frac{\Psi^2}{2} - \underline{U}.$$

Thus, the investor's expected payoff and the expected surplus under asymmetric information are, respectively,

$$EB^f = 1 + \frac{\Psi^2}{2} - \underline{U}, \tag{6.18}$$

and

$$S^f = EB^f + \underline{U}. \tag{6.19}$$

A comparison between the surplus generated by the franchise contract and that of the second-best solution (Corollary 2) reveals that

$$S^* > S^f,$$

which implies that the franchise scheme is not even a second-best solution despite that it induces a level of joint innovation equal to that of the second-best contract (i.e.,

$e^f = e^* = \Psi$). As in the straight debt case, this occurs because whereas the full-franchise scheme leads to a corner solution in which only the entrepreneur undertakes all the innovation, the second-best financing rule induces a perfectly balanced solution that involves also a positive investor's innovation. In turn, this is a consequence of the fact that the full-franchise contract only provides high-powered incentives to the entrepreneur as a residual claimant, but not to the investor as claimant of a fixed and riskless payment.

To illustrate the suboptimality of a franchise contract, figures 13 and 14 present the results of comparative statics exercises performed on surplus with respect to γ and m, under the same parameter values of Subsection 4.2.[25] These figures suggest that whereas the gap of surplus between the second-best and franchise contract increases with the level of probability extremism of the project, it decreases with the probability skew towards failure.
⟨Insert Fig. 13 and 14 here⟩

7 Concluding Remarks

This article characterizes the optimal contract involved in the financing of a highly innovative project investment when both the investor and the entrepreneur undertake innovation and address a moral hazard problem. Our model delivers results that mainly contribute to the previous literature in three aspects.

First, our framework considers innovation as an endogenous variable, which, at the optimum, we show depends on the primitive characteristics of a return distribution that relates nonmonotonically observed profitability and unobserved innovation decisions. These characteristics include the degree of skew regarding success returns and failure rates, as well as the level in which returns and probabilities concentrate around extreme results. This allows us to predict the expected social value generated by an investment project according to how much ex ante its primitives approximate it to one of the three profiles characterized by evidence available for real-world highly innovative ventures. In addition, this formulation provides us with a rich analytical setup to explain and predict the impact of technological shocks (real or perceived) on the levels of innovation in high-tech sectors.

Second, our research offers a novel explanation to the frequent use of hybrid securities when financing high-tech firms based on two interconnected elements: (i) the nonmonotone statistical relationship between innovation and returns that potentially characterizes highly innovative ventures, and (ii) the need for providing high-powered

[25] Numerical exercises with other parameters are also available upon the authors' request.

incentives to both parties of the contract because of the double moral hazard setup. As a result, whereas the no MLRP assumption implies that what matters to incentivize innovation is to satisfy a given relationship between the rewards of only extreme returns, the need for balancing high-powered incentives implies that an optimal financial arrangement must always combine both outside and inside equity.

Third, our work contributes with an alternative explanation to the credit rationing that in practice affects more severely to highly innovative firms. While previous literature have emphasized the role played by insufficient collateral, our explanation to credit rationing relies rather than on the primitives of the projects. Specifically, we establish that ex ante profitable ventures, but with too high rates of failure or with too much probability mass concentrated on moderate returns, may however become unfunded when using conventional debt schemes.

In light of these three contributions, we argue that the model here developed constitute a suitable and novel starting point to analyze the venture capital financing. This is because it captures two features of this class of financial vehicles: (i) the financing of highly innovative entrepreneurial business with abnormally large, but unlikely, return success, and (ii) the venture capitalist's provision not only of funds, but also of effort by means of either management, advise or monitoring.

Nevertheless, in order to better account for venture capital financing, our framework should be extended in several directions. First, a natural extension is to incorporate staged-financing, as empirical evidence shows that funding conditional on milestones is typical in venture capital financing. Second, a more general functional form for joint innovation can be assumed such that the two parties' innovation levels may not necessarily be substitute between them (for instance, they may be complementary with each other), which may affect some of our results, especially those concerning the suboptimality of a straight debt scheme. Third, one can conjecture that the effects of venture capitalist's actions on the return distribution are different from those effects coming from the entrepreneur's actions. Consequently, other suitable extension is to assume that whereas the entrepreneur's actions can be interpreted as innovation leading to more profitable riskier ventures, the venture capitalist's actions can be interpreted as managerial effort and advise leading to more profitable ventures *without* increasing risk (i.e. an improvement in a first-order stochastic dominance sense). Finally, as for our credit rationing result, it would be interesting to analyze how our explanation based on the primitives of the projects undertaken by highly innovative firms interacts with more conventional explanations based on the insufficient collateral that the same class of firms typically exhibit.

8 Appendix

Proof of Proposition 1. We first characterize first-best innovation levels. From the problem posed in subsection 3.1, it is straightforward to see that these optimal innovation levels must maximize the social value of the project, and thus, they must solve the following program:

$$\underset{a,p\in[0,1]}{Max} \sum_{i=1}^{3} \pi_i^{(a,p)} x_i - \frac{p^2}{2} - \frac{a^2}{2}$$

$$s.t.$$

$$a + p \leq 1.$$

The FOCs of this problem imply that

$$a^{FB} = \Psi \quad (8.1)$$

$$p^{FB} = \Psi, \quad (8.2)$$

with Ψ defined according to equation (3.2). Hence, the optimal joint innovation level is given by

$$e^{FB} = 2\Psi.$$

Note that assumption (A2) guarantees that $\Psi \in (0, \frac{1}{2})$, and thereby, that a^{FB} and p^{FB} are interior solutions. In turn, this ensures that joint innovation e^{FB} is an interior solution as well.

We now characterize the two optimal schemes that achieve these first-best innovation levels. To this end, we work on the investor's program described by equations (4.1)-(4.5).

(i) A full-insurance contract for the entrepreneur implies that $w_i = w$ for all i. Substituting this financing rule into the binding participation constraint (4.2) yields

$$w(a) = \underline{U} + \frac{a^2}{2},$$

which, after substituting into the investor's objective function (4.1) and taking FOCs, can be easily proved that implements the first-best innovation levels described by equations (8.1) and (8.2). Thus, the optimal full-insurance financing rule can be written as

$$w^{FB}(a) = \begin{cases} \underline{U} + \frac{\Psi^2}{2} & if \ a = \Psi \\ \underline{U} - \varepsilon & otherwise \end{cases}$$

where $\varepsilon > 0$ represents a penalization.

(ii) A full-franchise contract for the entrepreneur implies that $x_i - w_i = F$ for all i, or equivalently, that $w_i = x_i - F$ for all i. Substituting this financing rule into the binding entrepreneur's participation constraint yields

$$F^{FB}(a,p) = \sum_{i=1}^{3} \pi_i^{(a,p)} x_i - \underline{U} - \frac{a^2}{2}.$$

By plugging this term into the investor's objective function and taking FOCs with respect to a and p, it can be demonstrated that this franchise contract implements the first-best innovation levels of equations (8.1) and (8.2). Hence, the optimal franchise financing rule is described by the following fixed payment from the entrepreneur to the investor:

$$F^{FB}(p) = \begin{cases} 1 + \frac{3}{2}\Psi - \underline{U} & \textit{if } p = \Psi \\ \tau & \textit{otherwise} \end{cases}$$

with $\tau < 0$.

Finally, notice that assumptions imposed on Ψ and $\underline{U}$ ensure that the entrepreneur's limited liability constraint (4.4) and the investor's participation constraint (4.3) are verified. In the case of the latter constraint, the upper bound imposed by assumption (A3) on the reservation utility guarantees that it is satisfied with inequality. To check that, we compute the optimal investor's expected payoff according to equation (3.3), such that

$$EB^{FB} = 1 + \Psi^2 - \underline{U} > 0,$$

which completes the proof. □

Proof of Proposition 2. Thanks to the first-order approach (Rogerson, 1985), in the optimal sharing rule program we can substitute constraint (4.9) with the FOC of the problem that solves the optimal innovation level for the entrepreneur.[26] This condition is given by

$$m\gamma w_1 - \gamma\,(1+m)\,w_2 + \gamma w_3 - a = 0. \tag{8.3}$$

Moreover, it is easy to verify that at the optimal contract the participation constraint is binding and thus

$$(a+p)\gamma\,[m w_1 - (1+m)\,w_2 + w_3] + w_2 - \frac{a^2}{2} = \underline{U}. \tag{8.4}$$

After combining (8.3) and (8.4), we obtain

$$w_2(a,p) = \underline{U} - \frac{a^2}{2} - ap. \tag{8.5}$$

After replacing (8.5) into (8.3), it follows that

$$w_3(a,p,w_1) = \frac{\gamma\,(1+m)\left(\underline{U} - \frac{a^2}{2} - ap\right) - m\gamma w_1 + a}{\gamma}. \tag{8.6}$$

[26] The concavity of π_i with respect to a and p allows us to apply the first-order approach.

Then, thanks again to the first-order approach, from the first order condition of (4.10) we get

$$m\gamma\left(x_1 - w_1\right) - \gamma\left(1+m\right)\left(x_2 - w_2\right) + \gamma\left(x_3 - w_3\right) - p = 0. \tag{8.7}$$

After substituting (8.3) into (8.7), and using the definition of Ψ given by equation (3.2), we obtain

$$p(a) = \Psi - a. \tag{8.8}$$

The substitution of (8.8) into (8.5) yields

$$w_2(a) = \underline{U} + \frac{a^2}{2} - a\Psi, \tag{8.9}$$

Similarly, substitution of (8.8) into (8.6) delivers

$$w_3(a, w_1) = \frac{a}{\gamma} - mw_1 + (1+m)\left(\underline{U} + \frac{a^2}{2} - a\Psi\right). \tag{8.10}$$

After plugging (8.8), (8.9) and (8.10) into the investor's objective function we obtain the program

$$\underset{a\in(0,1]}{Max}\ \Psi a - a^2 + \frac{\Psi^2}{2} - \underline{U} + x_2.$$

The FOC with respect to a yields

$$a^* = \frac{\Psi}{2}.$$

Using this result in (8.8), (8.9), and (8.10) we finally obtain

$$\begin{aligned} p^* &= \frac{\Psi}{2}, \\ e^* &= \Psi, \\ w_2^* &= \underline{U} - \frac{3}{8}\Psi^2, \end{aligned}$$

and

$$w_3^* = A - mw_1^*, \tag{8.11}$$

with

$$A \equiv \frac{\Psi}{2\gamma} + (1+m)(\underline{U} - \frac{3}{8}\Psi^2).$$

It is now simple to verify that given assumption (A3), both sharing w_2^* and term A are positive. Finally, applying the limited liability constraints for w_1^* and w_3^* on equation (8.11) implies that

$$w_1^* \in \left[0, \frac{A}{m}\right],$$

and

$$w_3^* \in [0, A] .$$

Finally, we confirm that the investor's participation constraint (4.8) is satisfied. This fact follows from computing the optimal investor's expected payoff according to equation (3.3), such that

$$EB^* = 1 + \frac{3}{4}\Psi^2 - \underline{U} > 0,$$

where the inequality holds because of the upper bound imposed by assumption (A3) over $\underline{U}$. □

Proof of Corollary 2. (i) It follows directly from comparing innovation levels established in propositions 1 and 2, and the fact that assumption (A2) guarantees that $\Psi > 0$.

(ii) The substitution of the optimal contract $(a, p, \{w_i\}_{i=1}^3)$ under symmetric information (Proposition 1) and asymmetric information (Proposition 2) into equation (3.3) yields, respectively, the values of EB^{FB} and EB^* indicated in the corollary. The inequality in favor of EB^{FB} holds because $\Psi > 0$.

(iii) The substitution of the optimal innovation pair (a, p) under symmetric information (Proposition 1) and asymmetric information (Proposition 2) into equation (3.4) delivers, respectively, the values of surpluses S^{FB} and S^* indicated in the corollary. The inequality in favor of S^{FB} is a direct consequence of part (ii) of this corollary. □

Proof of Corollary 3. Notice first that applying the chain rule in Corollary 2, we can conclude that the sign of the effect of a marginal change in a given parameter on optimal innovation and surplus levels is equal to the sign of the partial derivative of Ψ with respect to such parameter.[27] Thus, it suffices to center our analysis on this class of derivatives. Accordingly, it is straightforward to establish the following results:

(i)

$$\frac{\partial \Psi}{\partial \gamma} = \sigma(k - m) + (k - 1) > 0,$$

which is true by assumption (A2).

(ii)

$$\frac{\partial \Psi}{\partial k} = \gamma(1 + \sigma) > 0,$$

since $\gamma, \sigma > 0$.

(iii)

$$\frac{\partial \Psi}{\partial m} = -\gamma\sigma < 0.$$

(iv)

$$\frac{\partial \Psi}{\partial \sigma} = \gamma(k - m) \begin{cases} \geq 0 & \text{if } k \geq m \\ < 0 & \text{if } k < m \end{cases} .$$

[27] In the case of surplus this is true since assumption (A2) implies that $\Psi > 0$.

□

Proof of Corollary 4. From Corollary 2 it is easy to see that the gaps between first-best and second-best innovation and surplus levels also depend on Ψ. Hence, the same results in terms of comparative statics for e and S proved in Corollary 3 can immediately be extended to those gaps. □

Proof of Corollary 5. First, from equations (6.11) and (6.12), under the U-shaped scheme the investor's ex post payoff is represented by the following structure:[28]

$$x_i - w_i^* = \begin{cases} 1-\sigma-\frac{A}{1+m} & \text{if } i=1 \\ 1-\underline{U}+\frac{3}{8}\Psi^2 & \text{if } i=2 \\ k(1+\sigma)-\frac{A}{1+m} & \text{if } i=3 \end{cases}. \tag{8.12}$$

We have then to show that the proposed mixed security scheme yields the same payoff structure (8.12) for each state of nature.
(i) When $i=1$, the investor's payoff is given by

$$x_1 - \max\{(1+\delta_1)x_1, 0\}, \tag{8.13}$$

where the first term represents the payoff coming from the full equity stake and the second term, the payoff coming from a short position in a zero-strike-price call option over a stake $(1+\delta_1)$ of the equity. Since $\delta_1 > 0$ and $x_1 > 0$, payoff (8.13) becomes

$$-\delta_1 x_1,$$

which after substituting δ_1 and x_1 turns out to be

$$1-\sigma-\frac{A}{1+m} = x_1 - w^* < 0,$$

where the inequality holds because, by assumption in Case 1, $x_1 < w^*$. Since this payoff is negative, the investor must then infuse additional funds by an amount

$$\begin{aligned} -\left(1-\sigma-\frac{A}{1+m}\right) &= \left(\frac{A}{(1+m)(1-\sigma)}-1\right)(1-\sigma) \\ &= \delta_1 x_1. \end{aligned}$$

(ii) When $i=2$, from adopting the same line of reasoning as before, and since $\delta_2 > -1$ and $x_2 = 1$, it follows that the investor's payoff is given by

$$\begin{aligned} x_2 - \max\{(1+\delta_2)x_2, 0\} &= 1-\underline{U}+\frac{3}{8}\Psi^2 \\ &= x_2 - w_2^*. \end{aligned}$$

[28] It is, of course, before the innovation disutility and initial investment.

(iii) When $i = 3$, since $\delta_3 > -1$ and $x_3 = k(1+\sigma)$, it follows that the investor's payoff is given by

$$\begin{aligned} x_3 - \max\{(1+\delta_3)x_3, 0\} &= k(1+\sigma) - \frac{A}{1+m} \\ &= x_3 - w^*, \end{aligned}$$

which completes the proof. □

Proof of Corollary 6. As in proof of Corollary 5, we must demonstrate that the proposed hybrid financing scheme yields an investor's ex post payoff structure identical to that characterized by expression (8.12).

(i) When $i = 1$, the investor's payoff is solely characterized by her equity stake:

$$(1-\alpha)x_1.$$

After substituting α and $x_1 = 1 - \sigma$, this payoff becomes

$$1 - \sigma - \frac{A}{1+m} = x_1 - w^* \geq 0,$$

where the inequality holds because, by assumption in Case 2, $x_1 \geq w^*$.

(ii) When $i = 2$, the investor's payoff is given by

$$\left(\frac{1-\alpha+\phi_2}{1+\phi_2}\right)x_2,$$

where the term in brackets represents the total equity stake of the investor after an amount ϕ_2 of new shares are issued in her favor.[29] After replacing α, ϕ_2, and $x_2 = 1$, we obtain that the investor's payoff is

$$1 - \underline{U} + \frac{3}{8}\Psi^2 = x_2 - w_2^*.$$

(iii) When $i = 3$, by the same logic when $i = 2$ the investor's payoff is described by

$$\begin{aligned} \left(\frac{1-\alpha+\phi_3}{1+\phi_3}\right)x_3 &= k(1+\sigma) - \frac{A}{1+m} \\ &= x_3 - w^*, \end{aligned}$$

where the first equality holds after substituting α, ϕ_3 and $x_3 = k(1+\sigma)$. □

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